II. Solution by W. W. BEMAN, A. M., Professor of Mathematics, State University, Ann Arbor, Mich.

$$\left(\tan^{-1}x-\frac{x}{1+x^2}\right)\frac{d^2y}{dx^2}=\frac{2x}{(1+x^2)^2}\left(x\frac{dy}{dx}-y\right).$$

Writing the equation in the form

$$\frac{d}{dx}\left(\frac{\tan^2x}{x}\right)\cdot\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{1+x^2}\right)\cdot\frac{d}{dx}\left(\frac{y}{x}\right)$$

it is obvious that $y=\tan^{-1}x$ and $y=x$ are independent particular integrals. Hence the complete primitive is $y=c_1\tan^{-1}x+c_2x$.

Solutions of this problem were also received from *L. C. WALKER*, and *G. B. M. ZERR*.

126. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find the volume contained between the conical surface whose equation is $z=a-\sqrt{(x^2+y^2)}$, and the planes whose equations are $x=z$ and $x=0$ by the formula $\int\int\int dxdydz$. [Todhunter's *Integral Calculus.*]

Solution by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

The cone clearly extends from vertex (0, 0, a) towards $z=0$. Hence in

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} dxdydz$$

we have $z_1=x$; $z_2=a-\sqrt{(x^2-y^2)}$; $x=a-\sqrt{(x^2+y^2)}$.

$\therefore$ $y^2=a^2-2ax$, $x_1=0$, and for x_2 we have $y_1=y_2$. $\therefore$ $x=\frac{1}{2}a$.

$$\therefore \text{ Volume}=\int_0^{\frac{1}{2}a}\int_{y_1}^{y_2}[a-x-\sqrt{(x^2+y^2)}]dxdy$$

$$=\int_0^{\frac{1}{2}a}\left[(a-x)(y_2-y_1)-\frac{y}{2}\sqrt{(x^2+y^2)}-\frac{x^2}{2}\log y+\sqrt{(x^2+y^2)}\right]_{y_1}^{y_2}dx$$

$$=\int_0^{\frac{1}{2}a}\left[2(a-x)\sqrt{(a^2-2xa)}-\sqrt{(a^2-2xa)}(a-x)-\frac{x^2}{2}\log\frac{a-x+\sqrt{(a^2-2ax)}}{a-x-\sqrt{(a^2-2ax)}}\right]dx.$$

Put $2x=a\sin^2\varphi$.

$$\therefore\ V=\int_0^{\frac{1}{2}\pi}a\sin\phi\cos\phi d\phi\left[(1+\cos\phi)\frac{a^2}{2}\cos\phi-\frac{a^2}{8}\sin^4\phi\log\left(\frac{1+\cos\phi}{1-\cos\phi}\right)^2\right]$$

$$=\frac{a^3}{2}\int_0^{\frac{1}{2}\pi}\left[\sin\phi\cos^2\phi+\sin\phi\cos^4\phi-\sin^5\phi\cos\phi\log\frac{1+\cos\phi}{\sin\phi}\right]d\phi$$

$$=\frac{a^3}{2}(\tfrac{1}{3}+\tfrac{1}{5})-\left[\frac{a^3\sin^6\phi}{12}\log\frac{1+\cos\phi}{\sin\phi}\right]_0^{\frac{1}{2}\pi}-\frac{a^3}{12}\int_0^{\frac{1}{2}\pi}\sin^6\phi\left(\frac{\cos\phi}{\sin\phi}+\frac{\sin\phi}{1+\cos\phi}\right)dx$$

$$=\tfrac{4}{15}a^3-\tfrac{1}{12}a^3\int_0^{\frac{1}{2}\pi}[\cos\phi\sin^5\phi+\sin^5\phi(1-\cos\phi)]d\phi=\tfrac{4}{15}a^3-\frac{a^3}{12}\cdot\frac{4.2}{5.3}=\tfrac{2}{9}a^3.$$

Similar solutions were received from *GEORGE LILLEY*, *LON C. WALKER*, *G. B. M. ZERR*, and *J. SCHEFFER*.

MECHANICS.

127. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Develop the Fourier Series to represent the temperature of a circular wire of uniform cross-section, in which the temperatures of the four quadrants are in order $t, 2t, 3t, 4t$.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

From Fourier's *Analytical Theory of Heat*, page 217, we get for the complete solution of the linear and varied movement of heat in a ring after a time T the following:

$$v=e^{-hT}\left[\frac{1}{2\pi}\int_0^{2\pi}f(x)dx+\frac{1}{\pi}\sum_{n=1}^{n=\infty}e^{-n^2kT}\sin nx\int_0^{\frac{1}{2}\pi}\sin nxf(x)dx\right.$$

$$\left.+\frac{1}{\pi}\sum_{n=1}^{n=\infty}e^{-n^2kT}\cos nx\int_0^{\frac{1}{2}\pi}\cos nxf(x)dx\right],$$

$$\text{where } f(x)=\frac{1}{2\pi}\int_0^{2\pi}f(x)dx+\frac{1}{\pi}\sum_{n=1}^{n=\infty}\sin nx\int_0^{2\pi}\sin nxf(x)dx$$

$$+\frac{1}{\pi}\sum_{n=1}^{n=\infty}\cos nx\int_0^{2\pi}\cos nxf(x)dx.$$

$$\int_0^{2\pi}f(x)dx=t\int_0^{\frac{1}{2}\pi}dx+2t\int_{\frac{1}{2}\pi}^{\pi}dx+3t\int_{\pi}^{\frac{3}{2}\pi}dx+4t\int_{\frac{3}{2}\pi}^{2\pi}dx=5\pi t.$$

$$\int_0^{2\pi}\sin nxf(x)dx=t\int_0^{\frac{1}{2}\pi}\sin nxdx+2t\int_{\frac{1}{2}\pi}^{\pi}\sin nxdx+3t\int_{\pi}^{\frac{3}{2}\pi}\sin nxdx+4t\int_{\frac{3}{2}\pi}^{2\pi}\sin nxdx.$$

$$=\frac{t}{n}(\cos\tfrac{1}{2}\pi n+\cos\pi n+\cos\tfrac{3}{2}\pi n)-\frac{3t}{n}=-\frac{4t}{n},\text{ except when } n=4m.$$

$$\int_0^{2\pi}\cos nxfxdx=t\int_0^{\frac{1}{2}\pi}\cos nxdx+2t\int_{\frac{1}{2}\pi}^{\pi}\cos nxdx+3t\int_0^{\frac{3}{2}\pi}\cos nxdx+4t\int_{\frac{3}{2}\pi}^{2\pi}\cos nxdx$$